Electroweak Sector of the SM

Roger Wolf
23. April 2015
Schedule for Today

1. Review of Lie-Groups:
   - $U(1) \ & \ SU(2)$
   - (Non-) Abelian Gauge theories

2. Phenomenology of Weak Interaction

3. Sketch of the Electroweak Sector of the SM:
   - Left (Right)-handed States
   - Local $SU(2) \times U(1)$ Symmetry
   - Weinberg Rotation
Quiz of the Day

• Are normal normal rotation in $\mathbb{R}^3$ Abelian or non-Abelian?

• The W boson only couples to left-handed particles! Does the Z boson also couple only to left-handed particles?

• Are the following gauge boson self-couplings allowed: $zww$, $wwww$?
Recap from Last Time

Gauge Field Theories:

\[ \psi(\vec{x}, t) \rightarrow \psi'(\vec{x}, t) = e^{i\vartheta} \psi(\vec{x}, t) \]
\[ \overline{\psi}(\vec{x}, t) \rightarrow \overline{\psi}'(\vec{x}, t) = \overline{\psi}(\vec{x}, t) e^{-i\vartheta} \]

\[ \partial_\mu \rightarrow D_\mu = \partial_\mu - ieA_\mu \]
\[ D_\mu \rightarrow D'_\mu = D_\mu - i\partial_\mu \vartheta \]
\[ A_\mu \rightarrow A'_\mu = A_\mu + \frac{1}{e} \partial_\mu \vartheta \]

\[ F_{\mu\nu} \equiv [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu \]
\[ F_{\mu\nu} \rightarrow F'_{\mu\nu} = F_{\mu\nu} \]

\[ \mathcal{L} = \overline{\psi} (i \gamma^\mu D_\mu - m) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \]
Review of Lie-Groups

Marius Sophus Lie

(*17. December 1842, † 18. February 1899)
Unitary Transformations

\[ \psi(\vec{x}, t) \rightarrow \psi'(\vec{x}, t) = e^{i\vartheta} \psi(\vec{x}, t) \]

- \( U(1) \) is a group of unitary transformations in \( \mathbb{R}^n \) with the following properties:
  \[ \mathbf{G} \in U(n), \quad \mathbf{G}^\dagger \mathbf{G} = \mathbb{I}_n, \quad \text{det} \mathbf{G} = \pm 1 \]

- Splitting an additional phase from \( \mathbf{G} \) one can reach that \( \text{det} \mathbf{G} = 1 \):
  \[ U(n) = U(1) \times SU(n) \]

\[ \text{det} \mathbf{G} = \pm 1 \quad \text{(Unitary Transformations)} \]
\[ \text{det} \mathbf{G} = +1 \quad \text{(Special Unitary Transformations)} \]
Infinitesimal → Finite Transformations

- The $SU(n)$ can be composed from infinitesimal transformations with a continuous parameter $\vartheta \in \mathbb{R}$:

$$G_{\text{finite}} = \mathbb{I}_n + iv_{\text{finite}} t \ (v_{\text{finite}} \in \mathbb{R}, \ t \in \mathcal{M}(n \times n))$$

$$G_{\text{finite}} = \left(\mathbb{I}_n + i\frac{v_{\text{finite}}}{m} t\right)^m \xrightarrow{m \to \infty} e^{iv_{\text{finite}} \cdot t} \quad t \text{ generators of } G.$$

- The set of $G$ forms a Lie-Group.

- The set of $t$ forms the tangential-space or Lie-Algebra.
Properties of $t$

- **Hermitian:**
  \[ G^\dagger G = I_n \]
  \[ = (I_n - i\vartheta t^\dagger) (I_n + i\vartheta t) = I_n + i\vartheta (t - t^\dagger) + O(\vartheta^2) \]
  \[ t = t^\dagger \]

- **Traceless** (example $SU(n)$):
  \[ \det G = \det (I_n + i\vartheta t) \]
  \[ = 1 + i\vartheta \text{Tr}(t) + O(\vartheta^2) \]
  \[ \Rightarrow \text{Tr}(t) = 0 \]

- **Dimension of tangential space:**
  \[
  \begin{pmatrix}
  * & * & * & * & * & * \\
  * & * & * & * & * & * \\
  * & * & * & * & * & * \\
  * & * & * & * & * & * \\
  * & * & * & * & * & * \\
  * & * & * & * & * & * 
  \end{pmatrix}
  \]
  - $n$ real entries in diagonal.
  - $\frac{1}{2} \cdot n(n - 1)$ complex entries in off-diagonal.
  - $-1$ for $SU(n)$ for det req.

- **$U(n)$ has $n^2$ generators.**
- **$SU(n)$ has $(n^2 - 1)$ generators.**
Examples that appear in the SM ($U(1)$)

- $U(1)$ Transformations (equivalent to $O(2)$):
  - Number of generators: $1^2 = 1$  
    **NB:** what is the Generator?
Examples that appear in the SM ($U(1)$)

- $U(1)$ Transformations (equivalent to $O(2)$):
- Number of generators: $1^2 = 1$  
  **NB:** what is the Generator?  
  The generator is 1.
Examples that appear in the SM ($SU(2)$)

- *$SU(2)$ Transformations* (equivalent to $O(3)$):
  - Number of generators: $(2^2 - 1) = 3$ i.e. there are 3 matrices $\{t_j\}$, which form a basis of traceless hermitian matrices, for which the following relation holds:

$$G = e^{i \sum_{j=1}^{3} \vartheta_j t_j}$$

- Explicit representation:

$$t_j = \frac{1}{2} \sigma_j \quad (j = 1 \ldots 3)$$

(3 Pauli Matrices)

$$[t_i, t_j] = i \epsilon_{ijk} t_k$$
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    \]
    \[ (3 \text{ Pauli Matrices}) \]
    \[
    [t_i, t_j] = i \epsilon_{ijk} t_k
    \]
  - algebra closes.
  - structure constants of $SU(2)$.
Non-Abelian Symmetry Transformations

• Example $O(3)$ (90° rotations in $\mathbb{R}^3$):

  switch $z$ and $y$: 

    1  2
    3  4
Non-Abelian Symmetry Transformations

- Example $O(3)$ (90° rotations in $\mathbb{R}^3$):
Non-Abelian Symmetries Transformations

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  switch $z$ and $y$: 

  cyclic permutation:
Non-Abelian Symmetries Transformations

- Example $O(3)$ (90° rotations in $\mathbb{R}^3$):

  switch $z$ and $y$:

  cyclic permutation:
Examples that appear in the SM ($SU(3)$)

- $SU(3)$ Transformations (equivalent to $O(4)$):
  - Number of generators: $(3^2 - 1) = 8$ ($\rightarrow$ 8 Gell-Mann Matrices)
Abelian vs. Non-Abelian Gauge Theories

**Abelian:**

\[
\begin{align*}
\psi(\vec{x}, t) &\rightarrow \psi'(\vec{x}, t) = e^{i\vartheta} \psi(\vec{x}, t) \\
\overline{\psi}(\vec{x}, t) &\rightarrow \overline{\psi}'(\vec{x}, t) = \overline{\psi}(\vec{x}, t) e^{-i\vartheta} \\
\partial_\mu &\rightarrow D_\mu = \partial_\mu - ieA_\mu \\
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F_{\mu\nu} &\equiv [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu \\
F_{\mu\nu} &\rightarrow F'_{\mu\nu} = F_{\mu\nu} \\
\mathcal{L} &= \overline{\psi} (i\gamma^\mu D_\mu - m) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}
\end{align*}
\]

**Non-Abelian:**

\[
\begin{align*}
\psi(\vec{x}, t) &\rightarrow \psi'(\vec{x}, t) = e^{i\vartheta a} t_a \psi(\vec{x}, t) \\
\overline{\psi}(\vec{x}, t) &\rightarrow \overline{\psi}'(\vec{x}, t) = \overline{\psi}(\vec{x}, t) e^{-i\vartheta a} t_a \\
\partial_\mu &\rightarrow D_\mu = \partial_\mu - igW_{\mu,a} t_a \\
D_\mu &\rightarrow D'_\mu = D_\mu - i [\vartheta a t_a, D_\mu] \\
W_\mu &\rightarrow W'_\mu = W_\mu + i [\vartheta a t_a, W_{\mu,a} t_a] + \frac{1}{g} \partial_\mu (\vartheta a t_a) \\
W_{\mu\nu} &\equiv [D_\mu, D_\nu] = \partial_\mu W_\nu - \partial_\nu W_\mu - ig [W_\mu, W_\nu] \\
W_{\mu\nu} &\rightarrow W'_{\mu\nu} = W_{\mu\nu} - i [\vartheta a t_a, W_{\mu\nu}] \\
\mathcal{L} &= \overline{\psi} (i\gamma^\mu D_\mu - m) \psi - \frac{1}{4} W_{a\mu\nu} W^{a\mu\nu}
\end{align*}
\]
The SM of Particle Physics

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
+ i F \bar{D} D + h.c. \\
+ \mathcal{L}_\text{Y} \mathcal{L}_\text{Y} + h.c. \\
+ |\partial_\mu \phi|^2 - V(\phi)
\]
Constituents and Interactions of the SM

18 free parameters

<table>
<thead>
<tr>
<th>Quarks</th>
<th>Fermions</th>
<th>Bosons</th>
<th>Force carriers</th>
</tr>
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<tbody>
<tr>
<td>$u$ up</td>
<td>$c$ charm</td>
<td>$\gamma$ photon</td>
<td>$U(1)$</td>
</tr>
<tr>
<td>$d$ down</td>
<td>$s$ strange</td>
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<td>$\times$</td>
</tr>
<tr>
<td>$b$ bottom</td>
<td></td>
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</tr>
<tr>
<td>$\ell$</td>
<td>Leptons</td>
<td>$g$ gluon</td>
<td>$SU(3)$</td>
</tr>
<tr>
<td>electron</td>
<td>$\nu_e$ electron neutrino</td>
<td>$8$ $SU(3)$</td>
<td></td>
</tr>
<tr>
<td>$\mu$ muon</td>
<td>$\nu_\mu$ muon neutrino</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau$ tau</td>
<td>$\nu_\tau$ tau neutrino</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 (left)</td>
<td>6 (right)</td>
<td>12 (Gauge fields)</td>
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45 (Fermion fields)
Constituents and Interactions of the SM

\[3 \cdot 6 (\ell. + r.)\]
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45 (Fermion fields)

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1 \(\rightarrow\) \(U(1)\)
3 \(\rightarrow\) \(SU(2)\)
8 \(\rightarrow\) \(SU(3)\)

12 (Gauge fields)
Phenomenology of Weak Interaction

- From the view of a high energy physics scattering experiment:
Change of Flavor & Charge
Parity Violation

- $W$ bosons couple only to **left-handed particles** (**right-handed anti-particles**):

- Maximally parity violating!
- Intrinsically violating CP as well!
Heavy Mediators

- Mediation by heavy gauge bosons:

\[ \frac{1}{Q^2} \]

\[ m_{\gamma} = 0 \]

\[ \frac{1}{Q^2 + m_W^2} \]

\[ m_W = 85.385 \pm 0.015 \text{ GeV} \]
The Model of Weak Interactions

Sheldon Glashow  (*5. December 1932)  

Steven Weinberg  (*3. Mai 1933)
**SU(2) Space of Weak Isospin**

- **Example:**
  
  \[ \psi_L = \begin{pmatrix} \nu \\ e \end{pmatrix} \]

  - **left-handed** \( e_L \) & \( \nu \) form **isospin doublet**.

  \[ e_R \]

  - **right-handed** \( e_R \) forms **isospin singlet**.

- **Left- & right-handed** components of fermions can be projected conveniently:

  \[
  e = e_L + e_R
  \]

  \[
  e_L = \left( \frac{1-\gamma^5}{2} \right) e \\
  e_R = \left( \frac{1+\gamma^5}{2} \right) e
  \]

  \[
  \bar{e}\gamma^\mu \left( \frac{1-\gamma^5}{2} \right) \nu = \bar{e}_L\gamma^\mu \nu_L
  \]

- **Lagrangian w/o mass terms** can be written in form:

  \[
  \mathcal{L}_0 = \bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + \bar{e}_R \gamma^\mu \partial_\mu e_R = \bar{e}_L \gamma^\mu \partial_\mu e_L + \bar{\nu} \gamma^\mu \partial_\mu \nu + \bar{e}_R \gamma^\mu \partial_\mu e_R
  \]
Covariant Derivative of $SU(2) \times U(1)$

Covariant derivative corresponding to $SU(2)$ acts on *isospin doublet* only.\(^1\)

$$\mathcal{L}_{IA}^{SU(2) \times U(1)} = \bar{\psi}_L \gamma^\mu \left( \partial_\mu + ig W_\mu^a t^a \right) \psi_L \cdots$$

\(^1\) Note a different sign convention.
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\]

\[
t^+ = t_1 + i t_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \text{(ascending operator)}
\]

\[
t^- = t_1 - i t_2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad \text{(descending operator)}
\]

\[
W_\mu^a t^a = \frac{1}{\sqrt{2}} (W_\mu^+ t^+ + W_\mu^- t^-) + W_\mu^3 t^3
\]

1) Note a different sign convention.
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\[ \mathcal{L}_{IA}^{SU(2) \times U(1)} = \overline{\psi}_L \gamma^\mu \left( \partial_\mu + i \frac{g'}{2} Y_L B_\mu + ig W_\mu^a t^a \right) \psi_L + \overline{e}_R \gamma^\mu \left( \partial_\mu + i \frac{g'}{2} Y_R B_\mu \right) e_R \]

Covariant derivative corresponding to $U(1)$ acts on *isospin doublet* (as a whole) and on *isospin singlet*.

1) Note a different sign convention.
Covariant Derivative of $SU(2) \times U(1)$

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\[
Q = I_3 + \frac{Y}{2} \quad \text{(Gell-Mann Nischijama)}
\]

Covariant derivative corresponding to $U(1)$ acts on *isospin doublet* (as a whole) and on *isospin singlet*.

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Covariant Derivative of $SU(2) \times U(1)$

Covariant derivative corresponding to $SU(2)$ acts on \textit{isospin doublet} only.\(^1\)

$$\mathcal{L}_{IA}^{SU(2) \times U(1)} = \bar{\psi}_L \gamma^\mu \left( \partial_\mu + ig' Y_L B_\mu + ig W_\mu^a t^a \right) \psi_L + \bar{e}_R \gamma^\mu \left( \partial_\mu + ig' Y_R B_\mu \right) e_R$$

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$Q = I_3 + \frac{Y}{2}$ (Gell-Mann Nishijama)

\(^1\) Note a different sign convention.
\( SU(2) \times U(1) \) Interactions

- **Charged current interaction:**

\[
\mathcal{L}_{IA}^{CC} = -\frac{g}{\sqrt{2}} \left[ \bar{\nu} \left( W^+_\mu \gamma^\mu \right) e_L + \bar{e}_L \left( W^-_\mu \gamma^\mu \right) \nu \right].
\]

- **Neutral current interaction:**

\[
\mathcal{L}_{IA}^{NC} = -\left( \frac{g}{2} W^3_\mu - \frac{g'}{2} B_\mu \right) (\bar{\nu} \gamma^\mu \nu) + \left( \frac{g}{2} W^3_\mu + \frac{g'}{2} B_\mu \right) (\bar{e}_L \gamma^\mu e_L) + \frac{g'}{2} B_\mu (\bar{e}_R \gamma^\mu e_R).
\]
**SU(2) \times U(1) Interactions**

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  \]

- From $t^+$:
  \[ e \to \nu \]

- From $t^-$:
  \[ \nu \to e \]

- $\propto Z_\mu$

- \[
  \begin{pmatrix}
  Z_\mu \\
  A_\mu
  \end{pmatrix}
  =
  \begin{pmatrix}
  \cos \theta_W & -\sin \theta_W \\
  \sin \theta_W & \cos \theta_W
  \end{pmatrix}
  \begin{pmatrix}
  W^3_{\mu} \\
  B_{\mu}
  \end{pmatrix}
  \]

- \[
  \sin \theta_W = \frac{g'}{\sqrt{g^2+g'^2}} \quad \cos \theta_W = \frac{g}{\sqrt{g^2+g'^2}}
  \]

  (Weinberg Rotation)
**SU(2) \times U(1) Interactions**

- **Charged current interaction:**

\[
\mathcal{L}^{CC}_{IA} = - \frac{g}{\sqrt{2}} \left[ \bar{\nu} (W^+_{\mu} \gamma^\mu) e_L + \bar{e}_L (W^-_{\mu} \gamma^\mu) \nu \right]
\]

Desired behavior: \( A_\mu \) couples to left- and right handed component of \( e \) in the same way!

- **Neutral current interaction:**

\[
\mathcal{L}^{NC}_{IA} = - \frac{\sqrt{g^2 + g'^2}}{2} Z_\mu (\bar{\nu} \gamma_\mu \nu) + \frac{\sqrt{g^2 + g'^2}}{2} \left[ (\cos^2 \theta_W - \sin^2 \theta_W) Z_\mu + 2 \sin \theta_W \cos \theta_W A_\mu \right] (\bar{e}_L \gamma_\mu e_L) \\
+ \frac{\sqrt{g^2 + g'^2}}{2} \left[ -2 \sin^2 \theta_W Z_\mu + 2 \sin \theta_W \cos \theta_W A_\mu \right] (\bar{e}_R \gamma_\mu e_R)
\]

What is the expression for \( e \) ?
**SU(2) × U(1) Interactions**

- **Charged current interaction:**
  \[
  \mathcal{L}_{IA}^{CC} = -\frac{g}{\sqrt{2}} \left[ \bar{\nu} (W_\mu^+ \gamma^\mu) e_L + \bar{e}_L (W_\mu^- \gamma^\mu) \nu \right]
  \]
  Desired behavior: $A_\mu$ couples to left- and right handed component of $e$ in the same way!

- **Neutral current interaction:**
  \[
  \mathcal{L}_{IA}^{NC} = -\frac{\sqrt{g^2 + g'^2}}{2} Z_\mu \left( \bar{\nu} \gamma^\mu \nu \right)
  \]
  \[
  + \frac{\sqrt{g^2 + g'^2}}{2} \left[ (\cos^2 \theta_W - \sin^2 \theta_W) Z_\mu + 2 \sin \theta_W \cos \theta_W A_\mu \right] (\bar{e}_L \gamma^\mu e_L)
  \]
  \[
  + \frac{\sqrt{g^2 + g'^2}}{2} \left[ -2 \sin^2 \theta_W Z_\mu + 2 \sin \theta_W \cos \theta_W A_\mu \right] (\bar{e}_R \gamma^\mu e_R)
  \]

What is the expression for $e$?  
\[
e = \sqrt{g^2 + g'^2} \sin \theta_W \cos \theta_W
\]
NB: Skewness of the $SU(2) \times U(1)$

- Gauge boson *eigenstates* of the symmetry do not correspond to the *eigenstates* of the IA:

$$
\begin{pmatrix}
Z_\mu \\
A_\mu
\end{pmatrix} =
\begin{pmatrix}
\cos \theta_W & -\sin \theta_W \\
\sin \theta_W & \cos \theta_W
\end{pmatrix}
\begin{pmatrix}
W_\mu^3 \\
B_\mu
\end{pmatrix}
$$

- Quark *eigenstates* of the $SU(2)$ do not correspond to the quark *eigenstates* of the $SU(3)$ (NB: which are the mass *eigenstates*):

$$
\mathcal{M}_{CKM} =
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
$$

$$
= \begin{pmatrix}
c_1 & s_1c_3 & s_1s_3 \\
-s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\
-s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta}
\end{pmatrix}
$$

$$
c_i = \cos \theta_i \ ; \ s_i = \sin \theta_i \ (i = 1\ldots3)
$$
Non-Abelian Gauge Structure of $SU(2)$

\[ \mathcal{L}_{\text{gauge}} = -\frac{1}{2} \text{Tr} \left( W^a_{\mu\nu} W^{a\mu\nu} \right) - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \]

\[ B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \]

\[ W_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + ig [W^a_\mu, W^a_\nu] \]

- Implies lepton universality of weak interaction. (→extensively tested @ LEP)

- Introduces:
  - Triple Gauge Couplings (TGC)
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  - No *CP* conservation;
  - No “*EWK symmetry* conservation”!
  - ...

Sneak Preview for Next Week

• Up to now the problem of mass has been completely ignored.

• Discuss how mass terms in the Lagrangian density will compromise local gauge symmetries.

• Discuss the dynamic generation of mass via spontaneous symmetry breaking.